The Whirlpool Galaxy M 51a, as seen by Hubble

Quantum Gravity

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First published in issue 32, 1969

The interactions that one observes in the physical universe are normally divided into four categories according to their botanical characteristics. In order of strength they are, the strong nuclear forces, electromagnetism, the weak nuclear forces and, the weakest by far, gravity. The strong and weak forces act only over distances of the order of 10^{-13} cm or less and so they were not discovered until this Century when people started to probe the structure of the nucleus. On the other hand electromagnetism and gravity are long range forces and can be readily observed. They can be formulated as classical, i.e. non quantum, theories. Gravity was first with the Newtonian theory followed by Maxwell's equations for electromagnetism in the 19th Century. However the two theories turned to be incompatible because Newtonian gravity was invariant under the Galilean group of transformations of inertial frames whereas Maxwell equations were invariant under the Lorentz group. The famous experiment of Michelson and Morley, which failed to detect any motion of the Earth through the luminiferous aether that would have been required to maintain Galilean invariance, showed that physics was indeed invariant under the Lorentz group, at least, locally. It was therefore necessary to formulate a theory of gravity which had such an invariance. This was achieved by Einstein in 1915 with the General Theory of Relativity.

General Relativity has been very successful both in terms of accurate verification in the solar system and in predicting new phenomena such as black holes and the microwave background radiation. However, like classical electrodynamics, it has predicted its own downfall. The trouble arises because gravity is always attractive and because it is universal i.e. it affects everything including light. One can therefore have a situation in which there is such a concentration of matter or energy in a certain region of space-time that the gravitational field is so strong that light cannot escape but is dragged back. According to relativity, nothing can travel faster than light, so if light is dragged back, all the matter must be confined to a region which is steadily shrinking with time. After a finite time a singularity of infinite density will occur.

General Relativity predicts that there should be a singularity in the past about 10,000 million years ago. This is taken to be the "Big-Bang", the beginning of the expansion of the Universe. The theory also predicts singularities in the gravitational collapse of stars and galactic nuclei to form black holes. At a singularity General Relativity would lose its predictive power: there are no equations to govern what goes into or comes out of a singularity. However when a theory predicts that a physical quantity should become infinite, it is generally an indication that the theory has broken down and has ceased to provide an accurate description of nature. A similar problem arose at the beginning of the Century with the model of the atom as a number of negatively charged electrons orbiting around a positively charged nucleus. According to classical electrodynamics, the electrons would emit electromagnetic radiation and would lose energy and spiral into the nucleus, producing a collapse of the atom. The difficulty was overcome

by treating the electromagnetic field and the motion of the electron quantum mechanically. One might therefore hope that quantisation of the gravitational field would resolve the problem of gravitational collapse. Such a quantisation seems necessary anyway for consistency because all other physical fields appear to be quantised.

So far we have had only partial success in this endeavour but there are some interesting results. One of these concerns black holes. According to the Classical Theory the singularity that is predicted in the gravitational collapse will occur in a region of space-time, called a black hole, from which no light or anything else can escape to the outside world. The boundary of a black hole is called the event horizon and acts as a sort of one way membrane, letting things fall into the black hole but preventing anything from escaping. However, when quantum mechanics is taken into the account, it turns out that radiation can "tunnel" through the event horizon and escape to infinity at a steady rate. The emitted radiation has a thermal spectrum with a temperature inversely proportional to the mass of the black hole. As the black hole emits radiation, it will loss mass. This will make it get hotter and emit more rapidly. Eventually it seems likely that the black hole will disappear completely in a tremendous final explosion. However the time scale for this to happen is much longer than the present age of the Universe, at least for black holes of stellar mass, though there might also be a population of much smaller primordial black holes which might have been formed by the collapse of irregularities in the early Universe.

One might expect that vacuum fluctuations of the gravitational field would cause "virtual" black holes to appear and disappear. Particles, such as baryons, might fall into these holes and be radiated as other species of particles. This would give the proton a finite lifetime. However it is difficult to discuss such processes because the standard perturbation techniques, which have been successful in quantum electrodynamics and Yang-Mills theory do not work for gravity. In the former theories one expands the amplitudes in a power series in the coupling constant. The terms in the power series are represented by Feynmann diagrams. In general these diverge but in these theories all the infinities can be absorbed in a redefinition or "renormalisation" of a finite number of parameters such as coupling constants as masses. However in the case of gravity, the infinities of different diagrams are different and so they would require an infinite number of renormalisation parameters whose values could not be predicted by the theory. In fact the situation is not really that much worse than with the so-called renormalisable theories since even with them the perturbation series is only asymptotic and does not converge, leaving the possibility of adding an arbitrary number of exponentially vanishing terms with undetermined coefficients.

The problem seems to arise from an uncritical application of perturbation theory. In classical general relativity it has been found that perturbation expansions around solutions of the field equations have only a very limited range of validity. One cannot represent a black hole as a perturbation of flat space-time yet this is what summing Feynmann diagrams attempts to do. What one needs is some approximation technique that will take into account the fact that the gravitational field and the space-time manifold can have many different structures and topologies. Such a technique has not yet been developed but we, at Cambridge, have been approaching the problem by studying the path integral approach formulation of quantum gravity. In this the amplitudes are represented by an integral over all metrics

$$\int D[g]\exp(-\hat{I}[g])$$

where $D[g]$ is some measure on the space of all metrics g and $\hat{I}[g]$ is the action of the metric g.

If the integral is taken over real physical metrics (that is, metrics of Lorentzian signature – + + +), the action I is real so the integral oscillates and does not converge. To improve the eigenvalues one does a rotation of 90° in the complex t-plane. This makes the metric positive definite (signature + + + + and the action I pure imaginary so that the integral is of the form

$$\int D[g]\exp(-I[g])$$

where $\hat{I} = -iI$. The Euclidean action $\hat{I}$ has certain positive definite properties.

One is thus led to the study of positive definite metrics (particularly solutions of the Einstein equations) on four-dimensional manifolds. If the manifolds are simply connected, their topology can be classified (at least up to homotopy) by two invariants, the Euler number as measuring the number of *holes* or *gravitational instantons* and the signature measures the difference between right-handed instantons and left-handed ones. It seems that the dominant contribution to the path integral comes from metrics with about one instanton per Planck volume 10^{-142} cm^{3}s. Thus space-time seems to be very highly curved and complicated on the scale of the Planck length 10^{-33} cm, even though it seems nearly flat on larger scales.

However we still do not have a proper scheme for evaluating the path integral. The difficulty lies in defining a measure $D[g]$ on the space of all metrics. In order to obtain a finite answer it seems necessary to make infinite subtractions and these leave finite undetermined remainders. There is a possible way of overcoming this difficulty which may come from an extension of General Relativity called supergravity. In this the spin 2 graviton is related to a spin 3/2 field and possibly fields of lower spin by anticommuting "supersymmetry" transformations. In these theories there is an equal number of bosons (integer spin particles) and fermions (half integer spin particles). The infinities that arise in the path integral from the integration over boson fields seem to cancel when the infinities that arise from the integration over the fermion fields, raising the hope that one could provide a proper mathematical definition of the path integral, maybe some limiting process.

Supergravity theories have another very desirable feature, they may unify gravity with the other interactions and particles in physics. In 1967 Salem and Weinberg proposed a unified theory of the electromagnetic and weak interactions. This has had considerable success in predicting experimental results though the final confirmation will have to wait for the next generation of particle accelerators. Nevertheless, it has given great stimulus to attempts to unify the strong, the weak and the electromagnetic interactions into a "Grand Unified Theory". A feature of such theories is that the complete unification is seen only at the very high energies of the order of 10^{19} Gev, at which quantum gravitational effects should become important. It may well be therefore that one will be able to achieve the unification only by incorporating gravity as well in a completely unified theory which would describe all of physics. This was the goal to which Einstein devoted the last thirty years of his life, without much success. The prospects look brighter now though it is still probably quite a long way off.